Energy balance of the bulk, Maxwellian electrons in spatially inhomogeneous negative-glow plasmas

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The energy balance of the Maxwellian (bulk) electrons is analyzed in spatially inhomogeneous negative-glow plasmas (NGP). The purpose is to give a comprehensive model which enables the electron temperature in the NGP to be predicted. Since the bulk of the electron distribution function (EDF) in the NGP is Maxwellian, the rates of many important plasma processes (e.g., ambipolar diffusion, recombination, stepwise processes), as well as the plasma potential, are controlled by the electron temperature. Knowledge of the electron temperature is thus of particular importance for such types of plasma. In order to calculate the EDF in the elastic energy range (slow electrons), a spatially inhomogeneous kinetic equation is employed, in which the electron-electron collision integral is fully incorporated. Owing to the complicated (nonlinear integro-differential) form of the electron-electron collision integral, the direct numerical solution of the full kinetic equation represents a difficult task. An efficient way to render the problem tractable consists in breaking the slow electrons up into two distinct groups, namely, the Maxwellian (trapped) and superthermal (untrapped) electrons. The parameters of the Maxwellian EDF can be found from the particle- and energy-balance equations. The superthermal EDF can be found from a reduced kinetic equation. The separation of the electron population into two groups allowed us to obtain an energy-balance equation for the Maxwellian (cold, trapped) electrons, which properly accounts for the most important physical mechanisms, such as heating due to Coulomb collisions with the superthermal (hot, untrapped) electrons. It is shown that the problem of finding the electron temperature in a weakly collisional NGP can be described correctly only at the kinetic level. In this situation, the use of the fluid approximation, in which the electron ensemble is treated in terms of its density and mean energy, results in a physically incorrect energy-balance equation. Furthermore, it is demonstrated that the ''nonlocal'' effects may be critical for the problem of finding the EDF in general, and the electron temperature in particular, so that the ''local'' (kinetic) models may also produce erroneous results. The principal terms in the energy-balance equation are identified, and this equation is simplified to allow a ready solution and implementation into a plasma code. The validity of the proposed model for predicting the electron temperature was confirmed by numerical calculations of the EDF from the full kinetic equation. The results of the paper can be applied to the NGP generated in direct-current glow discharges with planar or hollow cathodes, as well as to negative-glow-like plasmas, such as beam-generated and afterglow (decaying) plasmas. [S1063-651X(98)08811-4]

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I. INTRODUCTION

The negative-glow plasma (NGP) has received considerable attention in the past and in recent years. The NGP is generated in the cathode region of glow discharges with either cold [1–7] or hot [8–10] planar cathodes, and with hollow cathodes [11–15]. The most distinctive feature of the NGP (sometimes also called the *field-free* plasma) is that it is sustained by an ''external'' ionization source (''ionizer''). In a glow discharge, this source is due to fast electrons streaming into the NGP from the cathode-fall region. Externally injected beams of fast (neutral or charged) particles (e.g., electrons, atoms, see [16–18]) or (γ) radiation can be employed as such an ''ionizer.'' The NGP generally features enhanced ionization and excitation as compared with, e.g., the positive-column plasma. This makes the NGP-based discharges a useful tool in laser [14], lighting [8–10], and surface-processing [3,7] applications.

The direct-current glow discharges with either planar or hollow cathodes are the most common examples of discharges in which the NGP is generated. Another quite important type of plasma, which has properties similar to those of the NGP, is the afterglow (decaying) plasma. The pulsed (low-pressure) plasmas have been found to be very promising in plasma processing and strong efforts are now being made to model these plasmas (e.g., [19]). The presence of a superthermal electron population created in various processes involving excited particles (metastables) can greatly affect the discharge dynamics in the afterglow (see [20–25] for details). Also very close in nature to the NGP (and to the afterglow plasma) is the plasma of the Faraday dark space, which is where the fast electrons streaming from the cathode fall cannot reach, but where the electric field is weak, and where the current is transported by superthermal electrons [2]. To some extent, a negative-glow-like plasma is generated in the magnetic-multicusp discharges (e.g., [26]). The plasma in these devices is created by electrons emitted by a

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hot filament and accelerated in a narrow double layer surrounding the filament. Finally, the magnetron discharges (e.g., [27]) can be considered to consist partly of a NGP.

It is clear that in every model of the cathode region, and in every electrode-to-electrode model, one inevitably has to treat the NGP. In most of such simulation models, the electrons are divided into the *fast* and *slow* electrons [28,3,5,29,6,7,15]. The fast electrons, which are responsible for excitation and ionization of gas particles, are treated kinetically (Monte Carlo methods [3,11,15], the direct solution of the Boltzmann equation [30], the convective schemes [31], etc.). The slow electrons, which are not capable of exciting and ionizing, are treated using the fluid (hydrodynamic) approach. Usually the slow electrons are described in terms of the electron density n_e and temperature T_e, assuming that they have a Maxwellian distribution. While n_e is found from the continuity equations, different approaches exist to include the electron temperature in the model. Some works employ hydrodynamic energy-balance equations to predict T_e in the NGP (e.g., [32,33,3]). However, in most models, T_e is not calculated but used as an input parameter [28,8,4,9,29,11,7,15]. The value of T_e in the NGP is typically assigned to 1 eV (hereinafter, T_e is expressed in units of eV) regardless of the plasma conditions (e.g., [29,11,7,15]), or assumed to be close to the atom temperature T_a. And yet, not only can T_e be very different from T_a or 1 eV but it can also vary significantly depending on the plasma parameters, such as gas pressure, electron density, discharge geometry (e.g., [12,13,34,35]). Another quite common approach in modeling the negative-glow-like plasmas (e.g., afterglow, magnetic-multicusp discharges) is to use a ''local'' approximation (e.g., [36,26]). However, since these discharges typically operate at rather low gas pressures, the ''nonlocal'' effects, which are discussed in the present work, may manifest themselves most strongly, so that the ''local'' models may be in error. Since the majority of electrons in the NGP have a Maxwellian distribution, knowledge of the electron temperature is of primary importance. The rates of many important plasma processes (e.g., ambipolar diffusion, recombination, stepwise processes), as well as the plasma potential, in the NGP are controlled by T_e, and the use of *ad hoc* values of T_e can result in erroneous predictions of the electron density, metastable density, and so on.

The slow electrons (with kinetic energies $w \leqslant \epsilon^*$, where ϵ^* is the lowest excitation threshold of gas particles) in the NGP can be divided into two weakly interdependent groups, namely, the *Maxwellian* (bulk) electrons and the *superthermal* electrons (referred to as ''intermediate'' electrons in [2]). Such a division is possible since the characteristic energy, density, and time scales of these electron groups differ drastically. Indeed, the Maxwellian bulk is characterized by a high number density (n_e) and low mean energy (T_e); by contrast, the density of superthermal electrons, n_s, is significantly lower than n_e [$n_s \approx (10^{-4}-10^{-2})n_e$], and their mean energy, ϵ_s, is significantly higher than T_e ($\epsilon_s \approx \epsilon^*/2 \gg T_e$). Such differences result from the following equally important physical reasons: The electric field in the NGP is typically reversed (no direct field), or there may exist a direct field but its intensity is weak (e.g., [2,12]). As such the electron distribution function (EDF) exhibits relatively low mean energies and electron-electron Coulomb collisions become important, so that the bulk EDF tends to be Maxwellian with a low characteristic energy ($T_e < 1$ eV). Nonetheless, the entire EDF in the NGP is far from being Maxwellian. The efficient ionization by fast electrons ($w \gg \epsilon^*$) produces secondary electrons with energies appreciably higher than the mean energy of the Maxwellian bulk (cf. the positive-column plasma, where secondary electrons are produced mostly with low energies, $w \ll \epsilon^*$). Such energetic electrons (being ''too far'' from the Maxwellian bulk) do not have time to Maxwellize, and they form a ''hot'' tail of the EDF, which is essentially non-Maxwellian. The lifetime of a superthermal (untrapped) electron is determined by the fast processes of free diffusion to the wall, and of energy relaxation through electron-electron and electron-atom collisions. On the contrary, the lifetime of a bulk (trapped) electron is comparable to the characteristic times of the slow processes of ambipolar diffusion and recombination.

The bulk electrons in the NGP are trapped in the self-consistent (reversed) electric fields and so give practically no contribution to the electron current. As such, even though the superthermal (untrapped) electrons are much less numerous, they play several important roles in the discharge dynamics. They carry the electron current in the plasma and thus must be allowed for in calculations of the self-consistent electric fields in the plasma and wall sheaths. For example, drastic changes in the wall potential drop were observed and predicted in [23] due to the presence of a small amount of superthermal electrons, and scenarios when anomalously high wall potential drops may arise were analyzed in [22]. Furthermore, the superthermal electrons are essential to the energy balance of the bulk electrons. This is due to the absence of a direct (current-carrying) electric field in the NGP, which results in Coulomb collisions between the cold (bulk) and hot (superthermal) electrons being the primary energy gain mechanism for the cold electrons. Therefore consideration must be given to the existence of a superthermal electron population, which is not done in the traditional fluid approach. Since the superthermal electrons cannot be described in terms of two parameters, namely, n_e and T_e (fluid approach), they must be treated *kinetically*.

Already in the early studies of electron kinetics in the ionosphere (e.g., [37,38]) and afterglow (e.g., [20,21]) plasmas, the importance of superthermal electrons was recognized. More recently, the same conclusion has been drawn for plasmas produced by beams of charged particles (e.g., [18]) and for plasmas in hollow-cathode discharges (e.g., [12]). A much more significant number of works, however, deals with the cathode region of a glow discharge with planar electrodes. Bayle *et al.* [33] concluded that the only energy gain process for the electrons in the NGP is due to the work of electron pressure. Surendra *et al.* [3] allowed for heating of slow electrons ($w \leqslant \epsilon^*$) by fast electrons ($w > \epsilon^*$) only. Lawler *et al.* [1] have analyzed the power balance of the negative-glow electrons. Monte Carlo simulations were used to calculate the distribution of the ''hot'' electrons in the NGP. In the Monte Carlo simulations, a uniform nonreversed electric field (1.0 V/cm and 10 V/cm) was assumed and the ''cold'' trapped electrons were not included. An estimation of the power balance for a single set of discharge parameters was presented, which is based on the results of the Monte Carlo simulations and refined experi-

ments. It has been concluded in [1] that heating due to Coulomb collisions with the hot electrons is the dominant energy gain mechanism in the power balance of the cold electrons. Kolobov and Tsendin [2] have presented an analytic model of the cathode region of a short glow discharge. Their model incorporates an energy-balance equation for the trapped electrons based on a kinetic equation for the slow electrons in the ''nonlocal'' regime. It was pointed out in [2] that the problem of deriving the energy balance is complicated also by the fact that the value of T_e is sensitive to the shape of the EDF at total energies close to the wall potential energy.

In [12,34,13,14], a simplified model was used to calculate the electron temperature in the NGP of hollow-cathode discharges. Some assumptions of that model, however, remained to be validated by rigorous theoretical and numerical calculations. Such calculations necessitate the direct solution of a full kinetic equation, which represents an extensive computational task. So it is highly desirable to develop a comprehensive model for predicting the electron temperature in the NGP in a relatively simple manner, without solving a full kinetic equation. In the present paper such a model is presented and its results are compared with complete numerical calculations. In Sec. II the physical assumptions of the model are discussed. In Sec. III a spatially inhomogeneous kinetic equation is introduced, which allows one to calculate the slow EDF including electron-electron Coulomb interactions. An energy-balance equation for the Maxwellian electrons is introduced and limiting cases corresponding to the ''nonlocal'' and ''local'' regimes are analyzed in Sec. IV. In Sec. V numerical results are presented and compared with the theoretical predictions. Finally, Sec. VI gives the summary.

II. PHYSICAL ASSUMPTIONS OF THE MODEL

A model plasma discharge is considered here which approximates the NGP generated experimentally. Even though the results can be applied to different types of NGP, the NGP created by fast electrons is implied, as occurs in direct-current glow discharges with either planar or hollow cathodes. Such a plasma operates typically at gas pressures from 100's of mTorr to 10's of Torr and electron densities from $10^{10}\,\mathrm{cm}^{-3}$ to $10^{14}\,\mathrm{cm}^{-3}$. It is supposed that the NGP is surrounded by a dielectric wall defined as $\mathbf{r}\in\mathbf{S}_p$, where $\mathbf{r}$ is the spatial coordinate and $\mathbf{S}_p$ the plasma-boundary surface (e.g., $r=R$ in a cylindrical geometry, see Fig. 1). Other situations can also be explored by introducing appropriate boundary conditions, e.g., when the NGP is surrounded by an anode (metallic) surface [7], by a grid anode [34], or by a cathode surface [12,39]. The kinetic and energy-balance equations are of primary interest here and they are written down for a NGP of an arbitrary geometry (formulas in a vector notation). Although temporal evolution is allowed for in these equations, a steady-state plasma is assumed. The numerical simulations, comparisons with the theory, and analytic estimates are presented for a one-dimensional (1D) cylindrical geometry (see Fig. 1), but the results and conclusions can be extended to other geometries. This geometry approximates the NGP in the cathode region of a glow discharge with planar electrodes, in which the longitudinal dimension of the NGP exceeds its transverse dimension.

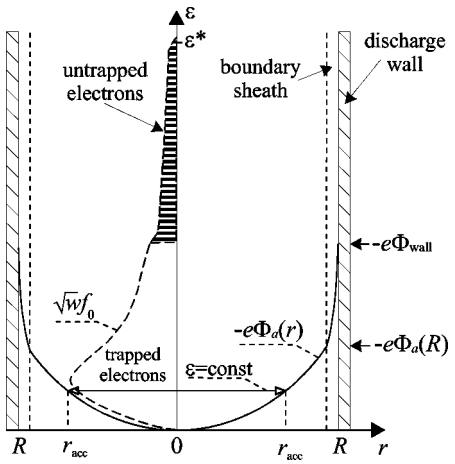

FIG. 1. Model discharge configuration of a cylindrical geometry. The EDF is shown schematically on the vertical energy axis. The shaded area represents the EDF of the untrapped electrons; $r=r_{\mathrm{acc}}(\epsilon)$ determine the turning points, between which an electron with a total energy ϵ bounces. The thickness of the boundary sheath is enlarged for clarity.

We limit ourselves to the case of a rare-gas discharge and consider only the slow electrons. The slow electrons include all those with kinetic energies $w\leqslant\epsilon^*$ (the elastic energy range). Thus all electrons (Maxwellian, superthermal, trapped, untrapped) considered in the paper are included among the slow electrons. In the absorbing-wall approximation $\nu^*\to\infty$ (where ν^* is the frequency of inelastic processes), we let the slow EDF vanish for energies $w\geqslant\epsilon^*$ (e.g., [2]); hence here $w=\epsilon^*$ is the maximum electron energy ($w_\infty=\epsilon^*$). Therefore we neglect inelastic processes involving ground-state atoms, as well as those involving metastables (see [24] for details). It is supposed that electrons appear in the elastic energy region through the action of an external ionizer (due to fast electrons with $w\gg\epsilon^*$). The fast electrons are not treated, and the rate of electron production (ionization) is assumed to be an input parameter. Also, processes involving metastables are discussed, by which superthermal electrons in the elastic energy range may appear (see [24] for details). Such processes can be of great importance in the NGP (see also [10]) since the metastable density is often comparable with, or exceeds, the electron density.

Finally, only the space-charge (ambipolar) electric field is assumed to exist in the plasma (no direct field). The reason is that the NGP with a spatial inhomogeneity generally features the presence of a *potential well* formed by the space-charge field in which the majority of electrons are trapped (see [2,12] for details). In this respect, it should be noted that calling the NGP the ''field-free'' plasma is, strictly speaking, not correct — there is always an electric field in the plasma, this field is self-consistent and such that the bulk electrons are trapped in the plasma [40].

III. INHOMOGENEOUS KINETIC EQUATION FOR THE SLOW ELECTRONS

The issues devoted to the nonlocal electron kinetics have been studied most thoroughly by Tsendin (e.g., [41]). Recently, these issues have attracted renewed attention, and several review papers have been presented by Tsendin

[42,40], Kolobov and Godyak [43], and Kortshagen *et al.* [44]. Here we only briefly outline some principal results and the reader is referred to these papers for further detail.

In order to calculate the slow EDF, a kinetic equation is to be derived. The following simplifications can be made. Let us suppose that the gas pressure is not too low (collisional regime), i.e., $\lambda \ll \Lambda$, where λ is the electron mean free path for momentum transfer and Λ the characteristic plasma dimension (e.g., $\Lambda \approx R/2$ for a cylindrical geometry). In the elastic energy range, the EDF can then be treated as being nearly isotropic due to the high rate of electron-atom collisions, which randomize quickly the directed motion of an electron. This enables the EDF to be expanded as $f(\mathbf{v},\mathbf{r},t) = f_0(v,\mathbf{r},t) + (\mathbf{v}/v)\cdot\mathbf{f}_1(v,\mathbf{r},t)$, where f_0 is the isotropic part of the EDF, $\mathbf{f}_1$ is its directed part ($|\mathbf{f}_1| \ll f_0$), and $\mathbf{v}$ is the electron velocity ($v=|\mathbf{v}|$). It is then highly convenient to write down the kinetic equation in variables $\mathbf{r}$ and total energy (kinetic energy plus potential energy) $\epsilon = w - e\Phi(\mathbf{r})$, where $\Phi(\mathbf{r})$ is the space-charge (ambipolar) potential [$-e\Phi(\mathbf{r}) \geqslant 0$] and $w = \frac{1}{2}mv^2$ the kinetic energy [41]. In order to simplify the equation for $\mathbf{f}_1$, and to allow for temporal evolution of f_0, one can assume that $\mathbf{f}_1$ is quasistationary (see [44] for details). In what follows, however, time dependence is neglected and the corresponding terms in the kinetic and energy-balance equations are retained for the sake of generality only. One can then write down the kinetic equation for $f_0(\epsilon,\mathbf{r})$ as

$$\frac{\partial f_0}{\partial t} = -\frac{1}{\sqrt{w}}\nabla\cdot\sqrt{w}\mathbf{J}_r + \frac{1}{\sqrt{w}}\frac{\partial}{\partial\epsilon}\sqrt{w}(J_{ee}+J_{ea}) + Q^+ + Q^*, \tag{1}$$

where

$$\mathbf{J}_r(\epsilon,\mathbf{r}) = \frac{1}{3}v\mathbf{f}_1 = -D_r\nabla f_0(\epsilon,\mathbf{r}) \tag{2}$$

is the electron flux in configuration space and $D_r = \frac{1}{3}\lambda v$ the electron diffusion coefficient;

$$J_{ee}(\epsilon,\mathbf{r}) = V_e f_0 + D_e\frac{\partial f_0}{\partial\epsilon} \tag{3}$$

is the electron flux in energy space due to electron-electron (e-e) Coulomb collisions, $V_e = 2\nu_e w A_1$, and $D_e = 2\nu_e w A_2$ with

$$A_1 = \frac{1}{n_e(\mathbf{r})}\int_0^{w(\mathbf{r})} f_0(w',\mathbf{r})\sqrt{w'}\,dw', \tag{4a}$$

$$A_2 = \frac{2}{3n_e(\mathbf{r})}\left(\int_0^{w(\mathbf{r})} f_0(w',\mathbf{r})w'^{3/2}dw' + w^{3/2}(\mathbf{r})\int_{w(\mathbf{r})}^{\infty} f_0(w',\mathbf{r})dw'\right), \tag{4b}$$

where $f_0(w,\mathbf{r})$ is expressed in terms of kinetic energy and $\nu_e = 4\pi n_e e^4\Lambda_C/(m^2v^3)$ is the frequency of e-e Coulomb collisions with Λ_C being the Coulomb logarithm;

$$J_{ea}(\epsilon,\mathbf{r}) = V_a f_0 + D_a\frac{\partial f_0}{\partial\epsilon} \tag{5}$$

is the electron flux in energy space due to electron-atom (e-a) elastic collisions with $V_a = \delta\nu_a w$ and $D_a = T_a V_a$, ν_a is the e-a collision frequency for momentum transfer, and $\delta = 2m/M$ is the fraction of electron energy lost in an elastic e-a collision. It should be mentioned that, in this (Fokker-Planck) notation, V_e (V_a) and D_e (D_a) represent, respectively, the dynamic-friction and diffusion coefficients in energy space due to e-e (e-a) collisions. The total dynamic-friction and diffusion coefficients can be introduced as $V_\epsilon = V_e + V_a$ and $D_\epsilon = D_e + D_a$, respectively. Similarly, it is possible to include electron-ion (e.g., [45]) and electron-molecule (e.g., [25]) collisions.

The term Q^+ in Eq. (1) represents the production of electrons by ionization due to the fast electrons (ionizer). Since the energy distribution of secondary electrons produced by ionization depends only slightly on the energy of the ionizing (fast) electrons, it is possible to separate the energy and spatial dependences in the ionization rate,

$$Q^+(w,\mathbf{r}) = Q^+(\mathbf{r})R^+(w), \tag{6}$$

where $Q^+(\mathbf{r})$ represents the number of ionizations per unit volume per unit time, and where the secondary-electron energy distribution $R^+(w)$ can be represented using an analytic approximation to the differential ionization cross section (Green's formula, see, e.g., [17]) to obtain

$$R^+(w) = \frac{2\epsilon^*}{\sqrt{w}(w+\epsilon^*)^2}. \tag{7}$$

Here, R^+ is normalized to unity in the elastic energy range: $\int_0^{\epsilon^*}\sqrt{w}R^+dw = 1$.

The last term in Eq. (1) represents the production of superthermal electrons in processes involving metastables. For the two most important of these processes, it can be written as

$$Q^* = \beta_P n_{\rm met}^2(\mathbf{r})R_P(w) + \beta_{\rm sup}n_{\rm met}(\mathbf{r})n_e(\mathbf{r})R_{\rm sup}(w), \tag{8}$$

where $n_{\rm met}$ is the metastable density; β_P (R_P) and $\beta_{\rm sup}$ ($R_{\rm sup}$) are the rate constants (energy spectra) corresponding, respectively, to the processes of Penning ionization and superelastic collisions [24]. As mentioned previously, the term Q^* can be very important for the NGP. However, for simplicity, in what follows this term is omitted, i.e., $Q^* = 0$ (see [24,12,14] for details).

The EDF in Eq. (1) is normalized according to

$$n_e(\mathbf{r}) = \int_{-e\Phi(\mathbf{r})}^{\infty}\sqrt{\epsilon + e\Phi(\mathbf{r})}f_0(\epsilon,\mathbf{r})d\epsilon. \tag{9}$$

This equation can also be regarded as a link between the electron-density and potential profiles, $n_e(\mathbf{r}) = n_e[\Phi(\mathbf{r})]$ (e.g., [41]). The electron-current density is

$$\mathbf{j}_e(\mathbf{r}) = e\int_{-e\Phi(\mathbf{r})}^{\infty}\sqrt{w}D_r(\epsilon,\mathbf{r})\nabla f_0(\epsilon,\mathbf{r})d\epsilon. \tag{10}$$

The boundary conditions for a cylindrical geometry (see Fig. 1) can be written in a compact form as

$$\left.\frac{\partial f_0(\epsilon,r)}{\partial r}\right|_{\substack{r=0\\ r=r_{\rm acc}(\epsilon)\\ r=R,\epsilon\leqslant -e\Phi_{\rm wall}}} =0. \tag{11}$$

Here, the first condition reflects the radial symmetry of the problem. The second condition in Eq. (11) states the absence of electron flux ($\epsilon<-e\Phi_{\rm wall}$) at the $r=r_{\rm acc}(\epsilon)$ boundary determined by $w(\epsilon,r)=0$ (see Fig. 1). The third condition in Eq. (11) is due to the fact that the electrons with total energies $\epsilon\leqslant -e\Phi_{\rm wall}$ are reflected by the space-charge potential drop, $-[\Phi_{\rm wall}-\Phi_a(R)]$, in the boundary sheath (which is assumed to be infinitely thin and fully collisionless for electrons). The boundary condition for the untrapped electrons ($\epsilon>-e\Phi_{\rm wall}$) has to account for the removal of electrons from the loss cone in velocity space due to the presence of the absorbing surface (wall) [46]. This results in the departure from an isotropic EDF and it can be taken into account approximately by applying the following boundary condition (e.g., [22]):

$$\left(v f_0(\epsilon,r)\frac{\delta\Omega}{4\pi}\right)\Bigg|_{r=R} =\left(-D_r(\epsilon,r)\frac{\partial f_0}{\partial r}\right)\Bigg|_{r=R}, \tag{12}$$

where

$$\delta\Omega\approx\pi\frac{\epsilon+e\Phi_{\rm wall}}{\epsilon+e\Phi_a(R)} \tag{13}$$

is the effective solid angle of the wall loss cone [46]. Note that this boundary condition, in conjunction with the boundary condition (11) at $r=R$, ensures continuity of $\partial f_0/\partial r$ at $\epsilon=-e\Phi_{\rm wall}$ since $\delta\Omega\to 0$ when $\epsilon\to -e\Phi_{\rm wall}$.

Once the kinetic equation and its boundary conditions are specified, a solution to this nonlinear partial-differential equation can be found numerically (see Sec. V). In the NGP, owing to the relatively low mean energy of electrons (cf., the positive-column plasma), the rate of *e-e* Coulomb collisions (at thermal energies) may exceed the rates of other (slow) processes (particularly, e.g., ambipolar diffusion and recombination) by many orders of magnitude. Since the *e-e* collision integral has a complicated (nonlinear integro-differential) form, the direct numerical solution of this equation represents a difficult task. However, it is known that when *e-e* Coulomb collisions dominate, the solution to the (homogeneous) kinetic equation is Maxwellian. Moreover, due to the efficient ionization by fast electrons, the EDF tail is raised significantly by superthermal electrons with essentially non-Maxwellian distribution; the superthermal part of the EDF occupies the major portion of the elastic energy region since the condition that $(4-7)T_e/\epsilon^*\ll 1$ is generally well satisfied. Hence, an obvious simplification that can be made is to break the EDF up into two parts, namely, the Maxwellian part $f_0^{(m)}$ for $w\lesssim w_m$ and the superthermal part $f_0^{(s)}$ for $w\gtrsim w_m$ (e.g., [21,18]), i.e.,

$$f_0(\epsilon,\mathbf{r})=f_0^{(m)}(\epsilon,\mathbf{r})+f_0^{(s)}(\epsilon,\mathbf{r}), \tag{14}$$

where the separation energy w_m can be estimated from the condition that $f_0^{(m)}(w_m)\approx f_0^{(s)}(w_m)$. Using analytic estimates of the EDF obtained in Secs. III A and III B below, one can obtain the following estimate: $w_m\approx T_e\ln(n_e/n_s)\approx\ln[\tau_{\rm amb}(2\nu_e+\delta\nu_a+\tau_s^{-1})]$, where $n_e\approx Q^+\tau_{\rm amb}$; $\tau_{\rm amb}$ and τ_s are, respectively, the times of ambipolar diffusion ($\tau_{\rm amb}\approx\Lambda^2/D_{\rm amb}$, where $D_{\rm amb}$ is the ambipolar diffusion coefficient) and of free diffusion of a superthermal electron to the wall ($\tau_s\approx\Lambda^2/D_r$). Under the studied discharge conditions, $w_m\approx(4-7)T_e$.

The separation in Eq. (14) enormously simplifies the problem since the shape of the bulk EDF is now known *a priori*, the parameters of which, i.e., the electron density and temperature, can be found from the corresponding particle- and energy-balance equations. In the situation when no *a priori* knowledge of the EDF (or its bulk part) is available, there is little point in deriving an energy (and particle) balance and so the full kinetic equation must be solved (cf., the positive-column plasma). As a first approximation, the superthermal EDF in Eq. (14) (being a solution to the inhomogeneous kinetic equation) can be obtained by ignoring *e-e* interactions between the superthermal electrons themselves and by considering only *e-e* interactions with the Maxwellian electrons. A limitation of this approach is that the details of the EDF in the transition region ($w\approx w_m$), from the Maxwellian distribution to the superthermal distribution, cannot be described accurately. However, since the Maxwellian EDF falls off rapidly with energy and the superthermal EDF is broad, this transition region is typically narrow (its width is of the order of T_e) and affects the solution only slightly. The full kinetic equation can then be simplified significantly by using a reduced (linear) *e-e* collision integral, which in its turn can be obtained assuming that the bulk EDF is Maxwellian with a given electron density and temperature. The following fits to coefficients $A_1(w,\mathbf{r})$ and $A_2(w,\mathbf{r})$ in Eqs. (4a) and (4b) can be used with good accuracy [47]:

$$A_1=\begin{cases} 0.385w/T_e, & w/T_e\leqslant 2.6\\ 1, & w/T_e>2.6\end{cases}\qquad A_2=T_eA_1. \tag{15}$$

The kinetic equation (1) involving these coefficients is no longer nonlinear integro-differential but linear differential and can be solved rather easily to find $f_0^{(s)}$ provided that n_e and T_e are known, i.e., $f_0^{(s)}=f_0^{(s)}(\epsilon,\mathbf{r},n_e,T_e)$. Furthermore, in order to find $f_0^{(s)}$ when $w/T_e\gg 1$, the energy-diffusion term with $D_\epsilon\partial f/\partial\epsilon$ can be ignored (i.e., $A_2=0$) to within corrections of the order of $T_e/w\ll 1$ (e.g., [18]).

The above simplifications enable one to take account of both spatial and energy relaxation of the slow electrons. It is, however, instructive to consider limiting cases in which the EDF is governed by either spatial motion or collisional effects. In these limiting cases, simple analytic estimates can be obtained, which can be useful in identifying the most important physical processes and in interpreting the numerical results. It follows from the kinetic equation (1) that the spatial scale, on which a significant change in electron energy takes place, is determined by the energy-relaxation length λ_ϵ [41,42]. Depending on the λ_ϵ/Λ ratio, the EDF formation regime can be either ''*nonlocal*'' or ''*local*'' (see

[41,42] for details). Since the EDF can be separated into the Maxwellian and superthermal parts [see Eq. (14)], one can obtain a rough estimate of λ_ϵ:

$$\lambda_\epsilon(w)=\begin{cases}\dfrac{\lambda}{\sqrt{\delta}}, & w\lesssim w_m\\ \dfrac{\lambda}{\sqrt{\delta}}\dfrac{1}{\sqrt{1+2\nu_e/\delta\nu_a}}, & w\gtrsim w_m.\end{cases} \tag{16}$$

Here, the difference between the λ_ϵ values for the Maxwellian ($w\lesssim w_m$) and superthermal ($w\gtrsim w_m$) electrons is due to the fact that no energy relaxation in *e-e* collisions occurs for the Maxwellian electrons ($J_{ee}=0$).

A. Nonlocal electron distribution function

In the limiting case when $\lambda_\epsilon\gg\Lambda$, the terms involving gradients of the EDF and potential are essential in the kinetic equation, and it is said that the EDF formation regime is ''*nonlocal*.'' This implies that the EDF in a given space region is determined not only by the plasma parameters of this region but also by those over the entire discharge volume. While an electron moves through the discharge volume, its total energy is (almost) conserved, i.e., $\epsilon=$const. Under nonlocal conditions, the electrons can be divided into two distinct groups, namely, the *trapped* (with $\epsilon\leqslant -e\Phi_{\rm wall}$) and *untrapped* (free) (with $\epsilon>-e\Phi_{\rm wall}$). The untrapped electrons escape quickly to the wall without experiencing significant changes in energy. The trapped electrons with a total energy ϵ can only move within a restricted (accessible) volume in the plasma $\mathbf{V}_{\rm acc}(\epsilon)$, which is determined by $\epsilon\leqslant -e\Phi(\mathbf{r})$ [e.g., for a cylindrical geometry, this volume is bounded by the $r=r_{\rm acc}(\epsilon)$ surface, see Fig. 1]. The trapped electrons consist of Maxwellian electrons [when $2\nu_e\gg\delta\nu_a$, see Eq. (19) below] and superthermal electrons with total energies $\epsilon\leqslant -e\Phi_{\rm wall}$; the untrapped electrons consist of those superthermal electrons with $\epsilon>-e\Phi_{\rm wall}$. Although the trapped electrons constitute the majority of electrons, the current that they transport is negligible, and the main part of the electron current is due to free diffusion of the untrapped electrons. In such a situation, the electron current $\mathbf{j}_e$ *cannot* be expressed in terms of the trapped EDF characteristics (and their spatial derivatives), namely, n_e and T_e, so that the kinetic treatment must be employed [2,42,40].

The trapped EDF, being approximately a function only of the total energy, can be expanded as $f_0^{(t)}(\epsilon,\mathbf{r})=f_{00}^{(t)}(\epsilon)+f_{01}^{(t)}(\epsilon,\mathbf{r})$, where $f_{01}^{(t)}\ll f_{00}^{(t)}$. Inserting this expression for $f_0^{(t)}$ into the kinetic equation (1) and performing spatial averaging over the accessible volume yields

$$\frac{\partial}{\partial\epsilon}\left(\langle\overline{\sqrt{w}V_\epsilon}\rangle f_{00}^{(t)}+\langle\overline{\sqrt{w}D_\epsilon}\rangle\frac{\partial f_{00}^{(t)}}{\partial\epsilon}\right)+\langle\overline{\sqrt{w}Q^+}\rangle=0, \tag{17}$$

where the spatial-diffusion term has vanished and $\langle\overline{X}\rangle(\epsilon)$ denotes averaging of $X(\epsilon,\mathbf{r})$ over the accessible volume $\mathbf{V}_{\rm acc}(\epsilon)$. Since the source term $\langle\overline{\sqrt{w}Q^+}\rangle$ can be neglected for the bulk, trapped electrons, the solution of Eq. (17) (the bulk EDF) can be written approximately as

$$f_{00}^{(t)}(\epsilon)=C_n\exp\left(-\int_0^\epsilon\frac{d\epsilon'}{T(\epsilon')}\right), \tag{18}$$

where C_n is the normalization constant and $T(\epsilon)=\langle\overline{\sqrt{w}D_\epsilon}\rangle/\langle\overline{\sqrt{w}V_\epsilon}\rangle$ is the characteristic temperature of an electron with an energy ϵ. In the case when *e-e* collisions are dominant ($2\nu_e\gg\delta\nu_a$) $T(\epsilon)\approx T_e$, and the bulk EDF takes the form of the Maxwell-Boltzmann distribution in terms of total energy:

$$f_0^{(m)}(\epsilon)=\frac{2}{\sqrt{\pi}}\frac{n_{e0}}{T_e^{3/2}}\exp(-\epsilon/T_e), \tag{19}$$

where n_{e0} is the value of the electron density at the position of zero reference potential (e.g., $r=0$ for a cylindrical geometry). Once the nonlocal EDF $f_{00}^{(t)}(\epsilon)$ is found, the small perturbation term $f_{01}^{(t)}(\epsilon,\mathbf{r})$ (which determines the electron flux in configuration space $\mathbf{J}_r$) can be obtained from the full kinetic equation (1); the equation for $f_{01}^{(t)}(\epsilon,\mathbf{r})$ then becomes (see [41] for details)

$$-D_r(\epsilon,r)\frac{\partial f_{01}^{(t)}}{\partial r}=\frac{1}{r}\int_0^r r'dr'\left\{\frac{\partial}{\partial\epsilon}\sqrt{w'}\left[V_\epsilon(\epsilon,r')f_{00}^{(t)}(\epsilon)+D_\epsilon(\epsilon,r')\frac{\partial f_{00}^{(t)}(\epsilon)}{\partial\epsilon}\right]+\sqrt{w'}Q^+(w',r')\right\},$$

where $w'=\epsilon+e\Phi(r')$ and the right-hand side of which is known provided that $f_{00}^{(t)}(\epsilon)$ is found from Eq. (17).

The untrapped electrons, being capable of escaping to the discharge wall, cannot be treated using the spatially averaged kinetic equation. Their nonlocal EDF can be obtained by ignoring the energy-relaxation terms in the kinetic equation (1); for a cylindrical geometry it becomes

$$f_0^{(s)}(\epsilon,r)=\int_r^R\frac{dr'}{r'D_r}\int_0^{r'}r''Q^+(w,r'')dr''+f_{\rm wall}^{(s)}, \tag{20}$$

where $f_{\rm wall}^{(s)}$ is the nonlocal EDF at the discharge wall, which can be found from the boundary condition (12). Assuming a spatially uniform ionization rate, $Q^+(\mathbf{r})=Q_0^+=$const, one can obtain

$$f_0^{(s)}(\epsilon,r)=Q_0^+R^+(w)\tau_s(w)[1-(r/R)^2+\beta_{\rm ref}(\epsilon)], \tag{21}$$

where $\beta_{\rm ref}(\epsilon)=\frac{2}{3}(\lambda/R)(\delta\Omega/4\pi)^{-1}$ is the effective reflection coefficient [see Eq. (12)].

B. Local electron distribution function

In the case when $\lambda_\epsilon\ll\Lambda$, electron energy relaxation takes place within a short distance, and it is said that the ''*local*'' regime is realized. The perturbation of the local EDF owing to spatial diffusion exists only within short distances (of the order of $\lambda_\epsilon\ll\Lambda$) from the plasma boundaries and the terms involving spatial gradients of $f_0(\epsilon,\mathbf{r})$ and $\Phi(\mathbf{r})$ can be neglected in the kinetic equation. In the local regime, in contrast with the nonlocal regime, it is not possible to discriminate between trapped and untrapped electrons. Provided that

$2\nu_e \gg \delta\nu_a$ at thermal energies, the bulk EDF is close to Maxwellian, which is now given in terms of kinetic energy as

$$f_0^{(m)}(w,\mathbf{r}) = \frac{2}{\sqrt{\pi}} \frac{n_e(\mathbf{r})}{T_e^{3/2}} \exp(-w/T_e). \tag{22}$$

Recalling that the ambipolar potential in the plasma can be calculated approximately as

$$\Phi_a(\mathbf{r}) = -(T_e/e)\ln n_e + \text{const}, \tag{23}$$

one can conclude that the Maxwellian EDFs in the nonlocal [Eq. (19)] and local [Eq. (22)] regimes coincide. Hence, the Maxwellian EDF is somewhat unique in these different regimes. Moreover, in the local regime, the Maxwellian EDF can be regarded as being ''nonlocal'' in the sense that it depends only on total energy ϵ, and thus, as in the nonlocal regime, the Maxwellian electrons can also be considered to be trapped. [Note that, due to the presence of superthermal electrons, a realistic ambipolar-potential profile may differ from the simplified profile of Eq. (23) [39].] As far as the superthermal ($w \gtrsim w_m$) electrons are concerned, their EDF in the local regime can be written approximately as

$$f_0^{(s)}(w,\mathbf{r}) = \frac{Q^+(\mathbf{r})}{\sqrt{w} V_\epsilon} \int_w^{\epsilon^*} \sqrt{w'} R^+(w') dw', \tag{24}$$

where the difference between the total and kinetic energies is neglected, as is the energy-diffusion term (see [18,39] for more details).

It is now appropriate to mention the following. The superthermal EDF in the nonlocal and local regimes (see Secs. III A and III B) can be calculated from a simplified kinetic equation by ignoring either the energy-relaxation terms or the spatial-diffusion term. The situation, however, is complicated by the fact that the electron mean free path λ varies (for the case of He, increases) with energy, which may result in different parts of the EDF being in different regimes. In this situation, the simplifications discussed in Secs. III A and III B may be invalid and the superthermal EDF can be found using the kinetic equation with a reduced *e-e* collision integral [Eqs. (3) and (15)], and/or neglecting the energy-diffusion term ($D_\epsilon = 0$). The resulting (linear) partial-differential equation can be solved rather easily by standard numerical techniques.

IV. ENERGY BALANCE OF THE MAXWELLIAN ELECTRONS

Having derived the kinetic equation, one can obtain the energy-balance equation by multiplying the kinetic equation (1) by the kinetic energy $w(\epsilon,\mathbf{r})$ and integrating it over energy. Integration over the entire energy range yields an equation for the mean energy of all electrons, $\mathcal{W}$. Nonetheless, there is only little sense in deriving an equation for $\mathcal{W}$ since the complete EDF, unlike the Maxwellian EDF, is not known *a priori*. Moreover, the physical mechanisms that govern formation of $\mathcal{W}$ and T_e are completely different (see Sec. V), and the energy balance of all electrons *cannot* give T_e (see Sec. IV C). In order to derive the energy balance of the Maxwellian electrons, the integration domain should be such that the EDF therein is close to Maxwellian. Thus the upper limit of integration ϵ^{int} can be chosen to be an energy where the EDF starts to deviate from Maxwellian (e.g., [25]), i.e., $\epsilon^{\text{int}} \approx \epsilon_m$, where ϵ_m is the total energy corresponding to w_m. Note that since the trapped EDF is very close to Maxwellian (i.e., $\epsilon_m \approx -e\Phi_{\text{wall}}$), the integration domain, notably in the nonlocal regime, can be chosen to be $\epsilon \leqslant \epsilon^{\text{int}} = -e\Phi_{\text{wall}}$. Hence, in this situation the energy balance of the Maxwellian electrons can be identified with that of the trapped electrons, and, for simplicity, one can use $\epsilon^{\text{int}} = -e\Phi_{\text{wall}}$ (see also [2]). The energy-balance equation for the Maxwellian electrons becomes

$$\frac{\partial}{\partial t}\left(\frac{3}{2} n_e T_e\right) = -\boldsymbol{\nabla}\cdot\mathbf{q}_e^{(m)} - \mathbf{E}\cdot\mathbf{j}_e^{(m)} - H_{ea}^{(-)} - H_{es}^{(-)} + H_{ea}^{(+)} + H_{es}^{(+)} + H_Q. \tag{25}$$

Here, the electron temperature, the heat flux, and current density of the Maxwellian electrons, respectively, are given by

$$T_e = \frac{2\langle w f_0\rangle}{3\langle f_0\rangle}, \quad \mathbf{q}_e^{(m)} = \langle w\mathbf{J}_r\rangle, \quad \mathbf{j}_e^{(m)} = e\langle \mathbf{J}_r\rangle, \tag{26}$$

where the energy average of a quantity X is

$$\langle X\rangle(\mathbf{r}) = \int_{-e\Phi(\mathbf{r})}^{\epsilon_m} \sqrt{\epsilon + e\Phi(\mathbf{r})} X(\epsilon,\mathbf{r}) d\epsilon.$$

The first term in Eq. (25) (divergence of the electron heat flux) describes energy transfer associated with electron (chaotic) motion. The second term in Eq. (25) results from diffusion in the electric field $\mathbf{E} = -\boldsymbol{\nabla}\Phi$. This term involves the current density of the Maxwellian electrons $\mathbf{j}_e^{(m)}$, which must not be mistaken for the total electron-current density $\mathbf{j}_e$ of Eq. (10). The total electron-current density $\mathbf{j}_e$ is to be equal to the ion-current density $\mathbf{j}_i$ at any point in the plasma, i.e., $\mathbf{j}_e = \mathbf{j}_i = eD_{\text{amb}}\boldsymbol{\nabla} n_e$. Typically, since the Maxwellian electrons are trapped, their current $\mathbf{j}_e^{(m)}$ is small in magnitude (i.e., $|\mathbf{j}_e^{(m)}|/|\mathbf{j}_e| \ll 1$) and also may have a complicated spatial distribution (determined by $f_{01}^{(t)}$). The last term in Eq. (25) is the heating term due to secondary electrons generated with $\epsilon \leqslant \epsilon_m$, $H_Q(\mathbf{r}) = \langle wQ^+\rangle$.

The terms in Eq. (25) due to *e-a* collisions are

$$H_{ea}^{(+)}(\mathbf{r}) = w^{3/2} V_a f_0|_{\epsilon_m} - \langle V_a f_0\rangle, \tag{27a}$$

$$H_{ea}^{(-)}(\mathbf{r}) = -w^{3/2} D_a \left.\frac{\partial f_0}{\partial\epsilon}\right|_{\epsilon_m} + \left\langle D_a \frac{\partial f_0}{\partial\epsilon}\right\rangle. \tag{27b}$$

The first terms in Eqs. (27a) and (27b) determine, respectively, the energy fluxes (in energy space) into and out of the energy interval $\epsilon \leqslant \epsilon_m$ (inward and outward fluxes, when $T_e > T_a$) due to collisions with atoms with a nonzero temperature T_a.

The condition for energy conservation in *e-e* collisions ($\langle wJ_{ee}\rangle = 0$ when $\epsilon^{\text{int}} = \epsilon^*$) allows the terms due to *e-e* collisions in Eq. (25) to be expressed as integrals over the EDF with energies above ϵ_m:

$$H_{es}^{(+)}(\mathbf{r}) = w^{3/2} V_e f_0 \big|_{\epsilon_m} + \int_{\epsilon_m}^{\infty} \sqrt{w} V_e f_0 \, d\epsilon, \tag{28a}$$

$$H_{es}^{(-)}(\mathbf{r}) = -w^{3/2} D_e \frac{\partial f_0}{\partial \epsilon}\bigg|_{\epsilon_m} - \int_{\epsilon_m}^{\infty} \sqrt{w} D_e \frac{\partial f_0}{\partial \epsilon} d\epsilon. \tag{28b}$$

Here, the second term in $H_{es}^{(+)}$ determines the heating rate of the Maxwellian electrons in *e-e* Coulomb collisions with the superthermal electrons. In such collisions an energy flux out of the energy interval $\epsilon \leqslant \epsilon_m$ may also be induced; this may result in cooling of the Maxwellian electrons, which is described by the second term in $H_{es}^{(-)}$. The first term in Eq. (28b) can be attributed to the energy flux out of the energy interval $\epsilon \leqslant \epsilon_m$ due to *e-e* interactions between the Maxwellian electrons within the potential well. This outward energy flux can also be identified with ''diffusion cooling'' of the Maxwellian electrons in *e-e* collisions. In the steady-state NGP, in the presence of an important superthermal population which transports almost all the electron current, this energy flux is small. However, this outward energy flux can be substantial in the energy balance of the Maxwellian electrons for an afterglow plasma at low pressures (and for discharges in heavy gases, such as Ar and Kr, see [22] for details), in which the sources of superthermal electrons are weak and the electron (particle) flux out of the energy interval $\epsilon \leqslant \epsilon_m$ is large (approximately equal to the ion flux j_i) [22].

In order to find T_e from the energy-balance equation in its general form of Eq. (25), knowledge of the complete EDF is still required. However, given the (superthermal) EDF with $\epsilon > \epsilon_m$, it is possible to calculate the heating term in Eq. (25) and thus reduce the problem to the solving of two coupled equations, namely, a simplified energy-balance equation for T_e and a reduced kinetic equation for $f_0^{(s)}$. Since the superthermal EDF depends only weakly on the electron temperature (see Secs. III A and III B), in practice, the equation for $f_0^{(s)}$ can be solved almost independently from the equation for T_e. Taking into account that $H_{es}^{(+)}/H_{es}^{(-)} \approx w/T_e \gg 1$ for $w > w_m$, and that the first term in $H_{es}^{(+)}$ can be neglected as compared with the second term (when $\epsilon_s \gg T_e$), the total heating rate, $H_{es} = H_{es}^{(+)} - H_{es}^{(-)}$, can be represented in a physically transparent form (e.g., [21,18]):

$$H_{es}(\mathbf{r}) = \int_{\epsilon_m}^{\infty} \sqrt{w} V_e f_0^{(s)} d\epsilon = Q^+(\mathbf{r}) \epsilon_{\mathrm{eff}}(\mathbf{r}), \tag{29}$$

where ϵ_{eff} characterizes the effective energy (in units of eV) transferred to the system of Maxwellian electrons in Coulomb collisions with the superthermal electrons created by ionization at a rate Q^+. In Eq. (29), account is taken of the fact that $f_0^{(s)} \propto Q^+$, as can be seen from Eqs. (21) and (24). (Note that an energy $\epsilon_{\mathrm{eff}}^*$ corresponding to Q^* can also be introduced in the same fashion [24,12,14].) Using similar considerations and assuming a Maxwellian EDF for $\epsilon \leqslant \epsilon_m$, it is also possible to derive a well-known expression for $H_{ea} = H_{ea}^{(-)} - H_{ea}^{(+)}$:

$$H_{ea}(\mathbf{r}) = n_e (1 - T_a/T_e) \langle w \, \delta\nu_a \rangle_m, \tag{30}$$

where $\langle \cdots \rangle_m$ denotes averaging over the Maxwellian EDF of Eq. (22). It can be shown next that, for the studied discharge conditions, the term $\mathbf{E} \cdot \mathbf{j}_e^{(m)}$ is small compared with H_{ea} and that, in the limiting cases, the term involving the heat flux can be simplified (see Secs. IV A and IV B).

We have now obtained a simplified energy-balance equation, an analysis of which allows us to draw some important conclusions. Using Eqs. (21) and (24), we can obtain an order-of-magnitude estimate

$$\epsilon_{\mathrm{eff}} \approx \frac{2\tilde{\nu}_e}{2\tilde{\nu}_e + \delta\tilde{\nu}_a + \tilde{\tau}_s^{-1}} \epsilon_s, \tag{31}$$

where $\tilde{\nu}_e = \nu_e(\epsilon_s)$, $\tilde{\nu}_a = \nu_a(\epsilon_s)$, $\tilde{\tau}_s = \tau_s(\epsilon_s)$. It can be seen from this estimate that the efficiency of energy transfer is governed by the frequency of Coulomb collisions, $\tilde{\nu}_e$, between the Maxwellian and superthermal electrons. In the case of strong *e-e* interaction, when $2\tilde{\nu}_e \gg (\delta\tilde{\nu}_a, \tilde{\tau}_s^{-1})$, the value of ϵ_{eff} is large, $\epsilon_{\mathrm{eff}} \approx \epsilon_s$ (a significant fraction of the superthermal energy is transferred to the cold electrons), and consequently T_e can be high. In the case of weak *e-e* interaction, when $2\tilde{\nu}_e \ll (\delta\tilde{\nu}_a, \tilde{\tau}_s^{-1})$, the value of ϵ_{eff} is small, $\epsilon_{\mathrm{eff}} \ll \epsilon_s$, and T_e can be close to the atom temperature, i.e., $T_e \approx T_a$. The situation when ϵ_{eff} is low can be realized in the nonlocal regime. In such a regime, the hot electrons diffuse quickly to the discharge wall and do not interact efficiently with the cold electrons. This is a manifestation of the nonlocal effects in the mechanism controlling the electron temperature (e.g., [22]), and attempts to calculate T_e in the local (kinetic) approximation may fail. For example, in [36], the T_e decay is modeled for a positive-column plasma at rather low gas pressures and, to obtain reasonable agreement with experiment, the energy of superthermal electrons ($\propto \epsilon_{\mathrm{eff}}^*$) had to be reduced from 11 eV to 0.5 eV (in Ne). This can be explained by the fact that the local model used in that work significantly overpredicts the $\epsilon_{\mathrm{eff}}^*$ values as compared with those expected at low gas pressures (nonlocal regime). One can also conclude from the above examples that the electron temperature can depend quite critically on the plasma parameters (gas pressure, electron density, discharge geometry, etc.), which must be allowed for in modeling the NGP.

It follows from the energy-balance equation (25) that, due to thermal conduction, equalization of electron temperature occurs within distances of the order of $\lambda_T \approx \lambda(T_e)/\sqrt{\delta}$. Hence, depending on the λ_T/Λ ratio, two different regimes of electron temperature formation can be realized as specified in the two following sections.

A. Nonlocal energy balance

In the case when $\lambda_T \gg \Lambda$, energy gained locally by the trapped electrons $[Q^+(\mathbf{r}) \epsilon_{\mathrm{eff}}(\mathbf{r})]$ is redistributed quickly through the accessible volume $\mathbf{V}_{\mathrm{acc}}$, which results in an isothermal Maxwellian electron population, i.e., a spatially uniform T_e: $T_e(\mathbf{r}) = \mathrm{const}$. Under nonlocal conditions, the entire plasma volume contributes to the electron temperature formation. The nonlocal energy-balance equation can be obtained by averaging the energy-balance equation (25) over the plasma volume $\mathbf{V}_p$, or directly from the nonlocal kinetic

equation (17); owing to the absence of the particle and heat fluxes of trapped electrons at the plasma boundary, this yields

$$\frac{\partial}{\partial t}\left(\frac{3}{2}\overline{n_e T_e}\right) = -\overline{H_{ea}} + \overline{H_{es}} + \overline{H_Q}, \tag{32}$$

where the volume average of a quantity $X(\mathbf{r})$ is

$$\overline{X} = \frac{1}{V_p}\int_{\mathbf{V}_p} X(\mathbf{r}) d^3\mathbf{r},$$

with $V_p = |\mathbf{V}_p|$. The steady-state nonlocal energy-balance equation (32) is thus an algebraic (though nonlinear) equation which can readily be solved.

The term in Eq. (32) due to *e-e* Coulomb collisions can be calculated using the nonlocal superthermal EDF of Eq. (21), and its crude estimate can be obtained as (see also [12])

$$\epsilon_{\text{eff}} \approx \int_{\epsilon_m}^{\infty} w 2\nu_e \tau_s \frac{2\epsilon^*}{(\epsilon^* + w)^2} d\epsilon, \tag{33}$$

from which it can be seen that the efficiency of energy transfer is now determined by the time of free diffusion to the wall τ_s and, since τ_s is short in the nonlocal regime, ϵ_{eff} of Eq. (33) is typically low.

B. Local energy balance

In the other limiting case when $\lambda_T \ll \Lambda$, the electron temperature is controlled by the local plasma parameters and thus can be spatially nonuniform. Spatial averaging of the energy-balance equation can no longer be performed and simplifications have to be made to calculate the term involving the electron heat flux. A traditional approximation (which is only valid in the local regime) is to represent the heat flux in terms of the electron temperature and its gradients (heat conduction and convection) (e.g., [45,48]):

$$\mathbf{q}_e^{(m)}(\mathbf{r}) = -K_e \boldsymbol{\nabla} T_e + \frac{5}{2}(\mathbf{j}_e/e)\, T_e, \tag{34}$$

where $K_e = K_e(n_e, T_e)$ is the electron thermal conductivity coefficient.

It is possible to show that the term involving $\mathbf{q}_e^{(m)}$ in Eq. (25) can be ignored provided that $\lambda_T \ll \Lambda$ (local energy balance). Hence, in the local regime, the energy-balance equation, again, becomes an algebraic (local) equation. In an intermediate situation, when the term involving $\mathbf{q}_e^{(m)}$ is not negligible in the energy-balance equation (25), the approximation (34) makes this equation a second-order ordinary differential equation. The boundary condition for this equation can be obtained from kinetic considerations (e.g., [49]). This condition establishes a link between the heat flux at the boundary and some known plasma parameters, and it suggests that the diffusion cooling that the electrons undergo in the boundary-sheath field equals the work that the electrons do to maintain this field (see also [48,32]).

In the local regime, the term $\mathbf{E}\cdot\mathbf{j}_e^{(m)}$ in Eq. (25) can be identified with the diffusion-cooling mechanism for the Maxwellian electrons; at elevated gas pressures (local and intermediate regimes), this term can well be neglected by comparison with H_{ea} (e.g., [22]). The term due to *e-e* collisions in Eq. (25) can be calculated using the local superthermal EDF of Eq. (24), and the following estimate can then be obtained (e.g., [18,12]):

$$\epsilon_{\text{eff}} \approx \int_{w_m}^{\infty} \frac{2\nu_e}{2\nu_e + \delta\nu_a} \frac{\epsilon^* - w}{\epsilon^* + w} dw, \tag{35}$$

from which it can be seen that ϵ_{eff} is typically high in the local regime.

It is interesting to mention some peculiarities of the electron kinetics in a hollow-cathode discharge (see [39,12] for details). An important feature of a hollow-cathode discharge is that all electrons therein are trapped by the strong cathode-fall fields ($\partial f_0/\partial r = 0$ at $r = R$). (Note, however, that the superthermal electrons can diffuse out along the direction of the external-current flow and carry the electron current to the anode, see [12] for details.) As a result of such a reduction in the ''diffusion time,'' the energy-transfer rate H_{es} may be large, and relatively high electron temperatures can exist (see also discussion in Sec. V). Typically, the nonlocal regime is realized in a hollow-cathode discharge, i.e., $\lambda_\epsilon > \Lambda$, and thus spatial averaging of the kinetic equation can be performed. The complete (not only the bulk part) EDF can be obtained from the nonlocal kinetic equation for the trapped electrons [Eq. (17)]. Interestingly, the superthermal EDF in this case will have a form of a ''local'' distribution [as of Eq. (24)], and the expression for ϵ_{eff} will be similar to the local one given in Eq. (35).

C. Comparison with the fluid approach

In the traditional fluid approach, the whole electron ensemble is replaced by an ''average'' electron, to which unique particle and energy fluxes are assigned. However, such an approach can hardly be used in the situation when there are two groups of electrons which exhibit distinctly different behaviors both in energy and in configuration spaces (see also [2,40]). For example, one of the major assumptions of the fluid approach is that the average lifetime of an electron should match the time of ambipolar diffusion τ_{amb}. However, in the NGP, even this assumption is violated for the majority of electrons. Indeed, since only a small fraction κ (where $\kappa \approx -e\Phi_{\text{wall}}/\epsilon^* \ll 1$) of the ionization flux Q^+ arrives within the energy interval $\epsilon \leqslant -e\Phi_{\text{wall}}$, which contains the majority of electrons, the average lifetime of a trapped electron is approximately $\kappa^{-1}\tau_{\text{amb}}$, which is much longer than τ_{amb}. The trapped electrons, notably in the nonlocal regime, do not participate in transport of the electron current and practically all the electron current is due to free diffusion of the superthermal electrons. Moreover, in the situation when two completely different (and almost independent) groups of electrons exist, the energy balance of the whole electron ensemble cannot be used to predict the mean energy of one of these groups, namely, T_e. In order to confirm this, it is instructive to compare the electron temperatures obtained from the kinetic considerations (T_e) with those obtained in the fluid approximation ($T_e^{(f)}$). For this purpose, we write the energy-balance equation obtained from

the kinetic equation (1) in the form of the well-known hydrodynamic energy-balance equation (e.g., [45,48,32]):

$$\frac{\partial}{\partial t}\left(\frac{3}{2}n_e T_e^{(f)}\right) = -\nabla\cdot\mathbf{q}_e - \mathbf{E}\cdot\mathbf{j}_e - H_{ea} + H_{\rm sec}, \qquad (36)$$

where the heat flux $\mathbf{q}_e$ can be found from Eq. (34), $H_{\rm sec}(\mathbf{r}) = \langle wQ^+\rangle$ is the heating rate by secondary electrons with $w \in [0,\epsilon^*]$, and H_{ea} is given by Eq. (30) (see also [16]). This equation is usually solved subject to the boundary condition which establishes equality of the heat flux and the thermal convection flux at the discharge wall: $q_e = \frac{5}{2}T_e^{(f)} j_e^{\rm th}$, where $j_e^{\rm th} = \frac{1}{4}n_e v_{\rm th}$ is the thermal flux with $v_{\rm th}$ being the thermal electron velocity.

The difference in nature between the energy-balance equation for the Maxwellian (trapped) electrons [see Eq. (25)] and the hydrodynamic energy-balance equation (36) is that the latter applies to the whole electron ensemble. As a consequence, Eq. (36) involves the total heat flux $\mathbf{q}_e$ and the total electron-current density $\mathbf{j}_e\,(\equiv\mathbf{j}_i)$ in the diffusion-cooling term $\mathbf{E}\cdot\mathbf{j}_e$; it does not involve heating by the superthermal electrons (the energy of an electron ensemble is conserved in *e-e* collisions, $\langle wJ_{ee}\rangle = 0$) but does involve heating by secondary electrons (which do not yet belong to the slow electron group). From the ''fluid'' point of view, the latter heating mechanism ($H_{\rm sec}$) is very efficient since the secondary electrons possess the full energy available to the system of slow electrons (typically, $H_{\rm sec} \gg H_{es}$); all the slow electrons are Maxwellian in the fluid approach and hence all energy $H_{\rm sec}$ goes into heating of these Maxwellian electrons, thus raising $T_e^{(f)}$. However, from the ''kinetic'' point of view, most of the energy $H_{\rm sec}$ is expended through diffusion to the wall and collisions with atoms and only its (small) fraction (i.e., H_{es}) is available to the Maxwellian (trapped) electrons. Furthermore, by contrast with Eq. (36), in the energy balance of the Maxwellian (trapped) electrons, the heating mechanisms (term H_{es}) are mostly due to the superthermal electrons, and heating by secondary electrons is negligible, $H_Q \ll H_{es}$. Finally, the term $\mathbf{E}\cdot\mathbf{j}_e$ can also be important in the hydrodynamic energy-balance equation (36) (the diffusion-cooling term), but this term is negligible in the energy-balance equation for the Maxwellian (trapped) electrons since the current of these electrons is practically zero ($|\mathbf{j}_e^{(m)}| \ll |\mathbf{j}_e|$). Hence, one can see that the physical mechanisms that govern formation of T_e and $T_e^{(f)}$ are entirely different. This can lead to completely different quantitative results. Indeed, a direct comparison presented in Sec. V demonstrates that the electron temperatures obtained from these energy-balance equations may differ by more than an order of magnitude. Thus, the fluid approach is altogether misleading in this situation (e.g., [2,42,40]) and cannot be used to describe correctly the energy balance of the Maxwellian (trapped) electrons (see also [35]).

V. NUMERICAL RESULTS AND COMPARISON WITH THE THEORY

In order to validate the theoretical results, the full kinetic equation (1) was solved numerically in a 1D cylindrical geometry, subject to the boundary conditions (11) and (12). The numerical scheme consisted in writing the kinetic equation in a discrete form and applying a central difference operator on a 2D (total-energy–radius) grid. The numbers of energy and radial cells were 300–800 and 50–100, respectively. To avoid numerical instabilities due to the strong nonlinearity of the *e-e* collision integral, the procedure proposed by Rockwood [50] was used. The idea of this procedure is to construct the matrix corresponding to the *e-e* collision integral in such a way as to preserve the three essential properties of this integral (see also [16,17]). Namely, when only the *e-e* collision integral is retained in the kinetic equation (1), (i) the steady-state solution must be Maxwellian, (ii) the density and (iii) energy of the electron gas must be conserved. It was verified numerically that starting with an arbitrary initial distribution, at every radial position, the three properties were satisfied within computational error. The resulting system of nonlinear algebraic equations (e.g., [50,16,17]) was then solved iteratively by adding an artificial time dependence to it and advancing in time [51]. Different iterative methods were tried and the predictor-corrector method [51] was found to give good convergence when using large time steps. Moreover, to speed up convergence, the time step was increased dynamically. (Computationally, at every odd-numbered iteration, the time step was doubled.) The precision of the finite-difference scheme was enhanced by using the Crank-Nicolson method [51]. Iterations were performed until the maximum relative difference between the EDF values coming from two successive iterations was less than a predetermined precision, which typically was less than 10^{-4}. However, no special effort to optimize the code was made and, in order to obtain a steady-state solution, it took about 2–10 CPU hours on a medium-performance workstation.

Calculations were carried out for a pure He discharge in a cylindrical tube of radius $R = 1$ cm by assuming a spatially uniform ionization rate, i.e., $Q^+(\mathbf{r}) = Q_0^+ = \text{const}$, and the gas at room temperature. The electron mean free path was approximated as $\lambda(w) = 4.5\times10^{-2}/p$ cm for $w \leqslant 3$ eV and $\lambda(w) = 2.6\times10^{-2}\sqrt{w}/p$ cm for $w > 3$ eV, where p is in units of Torr and w is in units of eV. Four cases are presented here as listed in Table I. Approximately one order-of-magnitude variation in gas pressure and electron density is explored. The electron density was varied by altering Q_0^+. The range of p and n_e is such that the nonlocal regime is realized in cases N1 and N2 and the intermediate regime is realized in cases N3 and N4. As an illustration, calculations of the λ_ϵ/Λ ratio from Eq. (16) are presented in Table I. One can see that the λ_ϵ/Λ ratio (at $r = 0$) is large at superthermal energies ($w = \epsilon^*/2 \gg w_m$) in cases N1 and N2, and that this ratio is of the order of unity in cases N3 and N4. Important for the energy balance, and for the Maxwellian part of the EDF ($w \lesssim w_m$), is the λ_T/Λ ratio, which is large at $p = 0.5$ Torr and close to unity at $p = 5$ Torr (see Table I). Discussion of the local regime (when, e.g., recombination in the plasma volume may become important) is beyond the present scope (see, e.g., [18]).

For the purposes of the present work, knowledge of the exact, self-consistent potential distribution is not necessary and a model potential was used:

$$\Phi(r) = \Phi_a(r) = \Phi_0 \ln[1-(1-\chi)(r/R)^2], \qquad (37)$$

where $r < R$ and $\chi = n_e(R)/n_e(0)$, with $n_e(0)$ and $n_e(R)$ be-

TABLE I. Numerical and theoretical results for four different plasma conditions for a He gas and tube radius $R=1$ cm. Here, p, Q_0^+, $\Phi_a(R)$, and $\Phi_{\rm wall}$ are the input parameters. The numerical values of $n_e(0)$, $j_e(R)$, T_e, and of the corresponding terms in the energy-balance equation (expressed in units of Q_0^+) are obtained from the EDFs computed from Eq. (1) and their complete definitions in Eqs. (26), (28a), (28b), etc. (see Sec. IV). The numbers in brackets following the numerical values of T_e represent those at the discharge center (superscript) and at the discharge wall (subscript). The theoretical values of T_e are obtained from Eq. (32) with the EDF being computed from the kinetic equation with a reduced *e-e* collision integral [Eq. (15)]; the numbers in brackets represent the T_e values obtained using the nonlocal EDF of Eq. (21).

	N1	N2	N3	N4
p (Torr)	0.5	0.5	5	5
Q_0^+ ($\mathrm{cm^{-3}\,s^{-1}}$)	3.9×10^{15}	1.2×10^{16}	3.4×10^{14}	1.7×10^{16}
$-\Phi_a(R), -\Phi_{\rm wall}$ (V)	0.22, 0.67	0.30, 1.0	0.22, 0.67	0.30, 1.2
$\lambda_\epsilon(\frac{1}{2}\epsilon^*)/\Lambda$, λ_T/Λ	17, 11	9.8, 11	1.9, 1.1	1.7, 1.1
$n_e(0)$ ($\mathrm{cm^{-3}}$), numerical	4.5×10^{10}	3.6×10^{11}	3.7×10^{10}	3.4×10^{11}
$j_e(R)$ ($\mathrm{mA\,cm^{-2}}$), numerical	0.30	0.92	0.022	0.99
$\mathcal{W}(0)$, $\mathcal{W}(R)$ (eV), numerical	0.092, 0.15	0.11, 0.13	0.045, 0.31	0.14, 0.15
$\overline{T_e}$ (eV), numerical	0.070 ($^{0.070}_{0.070}$)	0.093 ($^{0.093}_{0.093}$)	0.031 ($^{0.032}_{0.036}$)	0.082 ($^{0.087}_{0.079}$)
T_e (eV), theory	0.065 (0.065)	0.099 (0.11)	0.030 (0.029)	0.077 (0.087)
$\overline{T_e^{(f)}}$ (eV), fluid + Eq. (36)	2.6	1.4	0.18	0.40
$\overline{H_{es}}$ (eV), numerical	2.0×10^{-2}	1.7×10^{-1}	1.2×10^{-1}	6.3×10^{-1}
$\overline{H_{es}}$ (eV), theory + Eq. (15)	2.0×10^{-2}	1.3×10^{-1}	1.1×10^{-1}	5.6×10^{-1}
$\overline{H_{es}}$ (eV), theory + Eq. (21)	2.0×10^{-2}	1.7×10^{-1}	8.6×10^{-2}	7.2×10^{-1}
$\overline{H_{ea}}$ (eV), numerical	2.3×10^{-2}	1.8×10^{-1}	1.2×10^{-1}	6.4×10^{-1}
$\overline{H_{ea}}$ (eV), theory + Eq. (30)	2.3×10^{-2}	1.4×10^{-1}	1.1×10^{-1}	5.8×10^{-1}
$\overline{\mathbf{E}\cdot\mathbf{j}_e^{(m)}}$ (eV), numerical	2.3×10^{-5}	2.7×10^{-4}	5.3×10^{-4}	2.3×10^{-3}
$\overline{H_Q}$ (eV)	3.4×10^{-3}	1.1×10^{-2}	4.5×10^{-3}	1.6×10^{-2}

ing the electron density at the discharge center ($r=0$) and at the plasma boundary ($r=R$), respectively. In deriving Eq. (37), Eq. (23) is used and a parabolic electron-density profile is assumed. In each case presented, the value of Φ_0 was taken to be close to the value of T_e expected from previous simulations and model estimations. The value of χ was estimated from Bohm's criterion by equating the ion ambipolar flux j_i at the plasma-wall boundary to $n_e(R)v_B$ (where $v_B=\sqrt{T_e/M}$ is the Bohm velocity) [45], which gives $\chi\approx\lambda_i\sqrt{T_e}/(\Lambda\sqrt{T_a})$, where λ_i is the ion mean free path. Calculations indicate that $\chi=0.01-0.1$ in the range of the studied conditions. For simplicity, χ was fixed at 0.05 in all the cases considered. The value of the wall potential energy ($r=R$) was calculated as $\Phi_{\rm wall}=\Phi_a(R)+\Phi_{\rm sh}$, where $\Phi_{\rm sh}$ is the potential drop in the boundary sheath (see Fig. 1). Since we do not calculate the self-consistent potential distribution in the plasma and use a model profile [see Eq. (37)], it is not possible, nor necessary, to find the self-consistent values of $\Phi_{\rm sh}$, when the electron-current density $j_e(R)$ at the wall equals the ion-current density $j_i(R)$. Hence, we used $\Phi_{\rm sh}$ only to limit $j_e(R)$. By choosing the appropriate values of $\Phi_{\rm sh}$, we could obtain the ratio $j_e(R)/j_i(R)\lesssim2$ in the low-pressure cases, and $j_e(R)/j_i(R)\lesssim10$ in the high-pressure cases. Note that when $\Phi_{\rm sh}=0$, $j_e(R)/j_i(R)\sim10^2-10^3$ was observed.

The EDFs computed in cases N2 and N4 are presented in Fig. 2 at different radial positions. One can see in Fig. 2(a) (nonlocal regime) that the EDF of the trapped electrons ($\epsilon\leqslant-e\Phi_{\rm wall}$) is essentially spatially independent and that the EDF of the untrapped electrons ($\epsilon>-e\Phi_{\rm wall}$) exhibits significant spatial inhomogeneity. In case N4 [intermediate regime, see Fig. 2(b)], small deviations from spatial homogeneity for $\epsilon\lesssim-e\Phi_{\rm wall}$ take place. One can also see that the EDFs at thermal energies are very close to Maxwellian. It is interesting to note that in all the cases (even in the nonlocal regime), despite the losses at the wall, no marked depletion of the EDF was observed at total energies slightly above the wall potential energy, i.e., $\epsilon>-e\Phi_{\rm wall}$. This behavior of the EDF is due to the boundary condition (12) which takes into account that the untrapped electrons have to enter the wall loss cone before they can escape to the wall [see also Eq. (21)]. By imposing a zero boundary condition at $r=R$ ($f_0=0$ for $\epsilon>-e\Phi_{\rm wall}$, i.e., $\beta_{\rm ref}=0$), a well-pronounced depletion could be observed.

The radial dependences of the electron density, electron temperature, and mean energy are presented in Fig. 3 in cases N2 and N4. Some data (values of T_e and $\mathcal{W}$ at the discharge center and wall, etc.) in cases N1 and N3 are presented in Table I. One can see that in the nonlocal regime (cases N1 and N2) the spatial profile of T_e is, not surprisingly, almost uniform. In the intermediate regime (cases N3 and N4), only small variations of T_e with r are observed [see Fig. 3(b) and Table I]. By contrast, the mean energy $\mathcal{W}$ is spatially nonuniform and increases towards the discharge

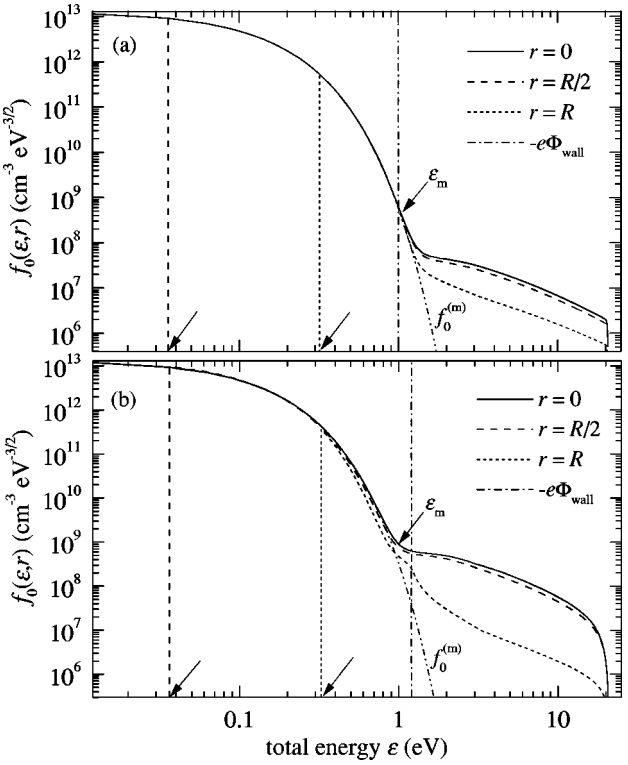

FIG. 2. EDFs computed from the full kinetic equation (1): (a) case N2, (b) case N4 (see Table I). The vertical lines represent the wall potential energy, $\epsilon = -e\Phi_{\rm wall}$. The arrows indicate the origins of the EDF $[w(\epsilon,r)=0]$ at different radial positions. Also shown are Maxwellian EDFs at $r=0$.

wall (see Fig. 3 and Table I). This can be explained by the low-energy (Maxwellian) part of the EDF being gradually removed, and the superthermal part becoming more pronounced, while approaching the discharge wall (see Fig. 2). Again, one can observe that the values of the mean energy

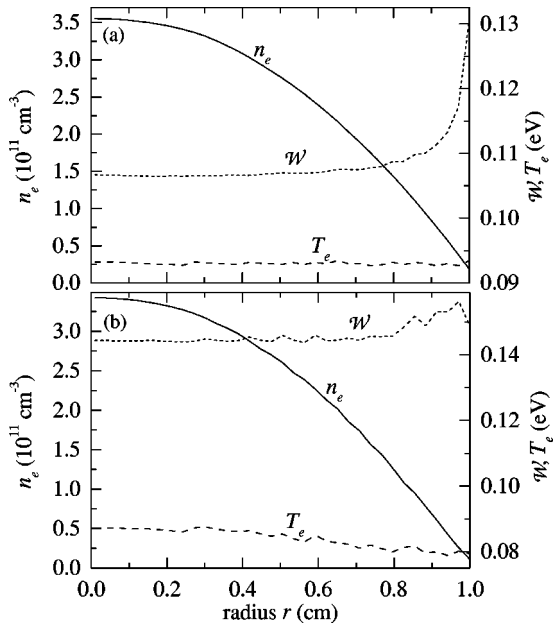

FIG. 3. Radial dependences of the electron density (n_e), electron temperature (T_e), and mean energy ($\mathcal{W}$) obtained numerically: (a) case N2, (b) case N4 (see Table I).

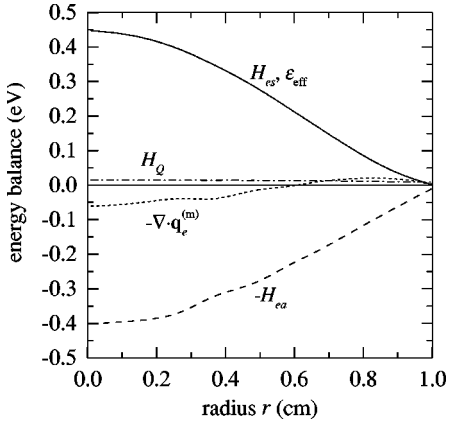

FIG. 4. Radial dependences of the terms in the energy-balance equation (25) obtained numerically in case N2. The data are normalized to Q_0^+. The term $\mathbf{E}\cdot\mathbf{j}_e^{(m)}$ is small and not plotted (see Table I).

and electron temperature are rather different, not only in spatial behavior, but also in magnitude (see Fig. 3 and Table I).

The terms (obtained numerically) in the energy-balance equation (25) as functions of the radial position are presented in Fig. 4. Data obtained for case N2 only are presented since these terms exhibit the same spatial behavior in the other studied cases as in case N2 (see Table I for their absolute values). For clarity, in Table I and Fig. 4, the terms in the energy-balance equation are normalized to Q_0^+ and are in units of eV; in this normalization, the values of H_{es} and $\epsilon_{\rm eff}$ coincide. One can see in Table I and Fig. 4 that H_{es} and H_{ea} are the two dominant terms and that the term due to heating by secondary electrons, H_Q, and the term $\mathbf{E}\cdot\mathbf{j}_e^{(m)}$, are small in all the cases. Recall that the contribution of the term involving $\mathbf{q}_e^{(m)}$ to the spatially averaged energy-balance equation is zero, i.e., $\overline{\nabla\mathbf{q}_e^{(m)}}=0$. The results of the complete numerical calculations demonstrate also that the simplifications of the terms H_{es} and H_{ea} made in Sec. IV (the neglect of $H_{es}^{(-)}$ by comparison with $H_{es}^{(+)}$, etc.) are well justified. In the calculations we used $\epsilon_m = -e\Phi_{\rm wall}$, and the results showed that varying ϵ_m (say within the range $[-25\%, +75\%]$) had little effect on the values of H_{es}, which thus confirms that the solution is not sensitive to the choice of ϵ_m.

A comparison between the values of T_e calculated numerically and those obtained from the present theoretical analysis is reported in Table I. In all the cases considered, the set of equations used to obtain T_e was the following: the nonlocal energy-balance equation (32) was employed to find T_e; H_{es} was obtained from Eq. (29), in which the superthermal EDF was calculated from the reduced kinetic equation [Eq. (1) with linear coefficients of Eq. (15)]; H_{ea} was calculated from Eq. (30). In order to check the applicability of the nonlocal approach, the nonlocal superthermal EDF of Eq. (21) was also used to calculate H_{es} of Eq. (29). One can see in Table I that the theoretical and numerical values of H_{es} are essentially in agreement in all the cases presented. Although the validity of the nonlocal approach is questionable in the intermediate regime, the numerical and theoretical values of T_e are also in close agreement in cases N3 and N4. One can observe in Table I that the energy-transfer rate in

the high-pressure cases (cases N3 and N4) is significantly larger than that in the low-pressure cases (cases N1 and N2). Despite this fact, T_e is not higher in cases N3 and N4 (in case N3, $T_e \approx T_a$), which is due to the higher rate of cooling through *e-a* collisions.

In order to make a direct comparison between the values of T_e obtained at the kinetic level with those predicted in the fluid approximation ($T_e^{(f)}$), the hydrodynamic energy-balance equation (36) was solved numerically. The results showed that the electron temperature is spatially uniform due to the high thermal conductivity. The spatially averaged values of $T_e^{(f)}$ from Eq. (36) are presented in Table I. One can see that the values of $T_e^{(f)}$ are significantly overestimated (by more than an order of magnitude in the nonlocal regime) as compared with those obtained from the full kinetic equation and from the energy-balance of the Maxwellian electrons. One of the reasons of such a large discrepancy is that, in the fluid approximation, the full available energy $H_{\rm sec}$ goes into heating of an ''average'' electron, whereas in reality only a small fraction of this energy is available to the Maxwellian electrons (see Sec. IV C). Hence it should be stressed again that the hydrodynamic energy-balance equation cannot be used to predict the electron temperature.

The present model in its simplified form has been applied to calculate the electron temperature in the NGP of hollow-cathode discharges of different geometries, and the results reported elsewhere [12,34,13] revealed good agreement with experiment. Moreover, calculations of the superthermal EDF from the reduced kinetic equation were found to be in close agreement with the EDFs measured by probes [39,13]. As mentioned in Sec. IV B, due to the large energy-transfer rate H_{es}, the values of T_e in the NGP of hollow-cathode discharges can be relatively high. For example, $T_e \approx 0.3$ eV at $p=3$ Torr in He and $n_e \approx 3.5\times 10^{11}\,\mathrm{cm}^{-3}$ in a cylindrical hollow-cathode discharge [12]. For modified hollow-cathode discharges, even higher T_e (up to 0.6 eV at low gas pressures) were measured and predicted [34,13]. Lawler *et al.* [1] observed lower values of T_e in the NGP of a glow discharge with planar electrodes, e.g., $T_e \approx 0.1$ eV at $p=3.5$ Torr in He, $n_e \approx 4\times 10^{11}\,\mathrm{cm}^{-3}$ and $0.846\ \mathrm{mA\,cm}^{-2}$ current density. It is appropriate to mention that the plasma conditions of that work and those in case N4 (see Table I) are somewhat similar, and the values of T_e observed in [1] and those predicted here in case N4 are rather close. We can also note that in heavier gases (such as Ne and Ar), which exhibit small *e-a* collision frequencies and have smaller δ factors, the electron temperature is expected to be higher than in He.

VI. SUMMARY

In summary, a model is presented that enables the electron temperature in the negative-glow plasma to be predicted in a simple manner. It is emphasized that knowledge of the electron temperature is crucial in modeling the NGP and that the dynamics of the NGP, particularly the energy balance of the (cold) Maxwellian electrons, is significantly affected by the presence of (hot) superthermal electrons. This fact, and the fact that the superthermal electrons must be described kinetically, imply that the electron temperature can be calculated properly only at the kinetic level. It is shown that in the situation when the electron ensemble consists of distinct groups (e.g., trapped, untrapped), the use of the fluid approximation results in a physically incorrect energy-balance equation, which produces quantitatively erroneous results. The results of the proposed model were validated by numerical solution of the full kinetic equation over a wide range of gas pressures and electron densities.

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[1] J. E. Lawler, E. A. D. Hartog, and W. N. G. Hitchon, Phys. Rev. A **43**, 4427 (1991).
[2] V. I. Kolobov and L. D. Tsendin, Phys. Rev. A **46**, 7837 (1992).
[3] M. Surendra, D. B. Graves, and L. S. Plano, J. Appl. Phys. **71**, 5189 (1992).
[4] M. Dalvie, S. Hamaguchi, and R. T. Farouki, Phys. Rev. A **46**, 1066 (1992).
[5] P. Bayle and A. Perrin, Phys. Rev. E **47**, 612 (1993).
[6] J. P. Boeuf and L. C. Pitchford, J. Phys. D **28**, 2083 (1995).
[7] A. Bogaerts, R. Gijbels, and W.-J. Goedheer, J. Appl. Phys. **78**, 2233 (1995).
[8] J. H. Ingold, Phys. Rev. A **43**, 3093 (1991).
[9] S. E. Coe and G. G. Lister, J. Appl. Phys. **71**, 4781 (1992).
[10] R. C. Wamsley, K. Mitsuhashi, and J. E. Lawler, Phys. Rev. E **47**, 3540 (1993).
[11] M. P. Alberta, J. Derouard, L. C. Pitchford, N. Ouadoudi, and J. P. Boeuf, Phys. Rev. E **50**, 2239 (1994).
[12] R. R. Arslanbekov, A. A. Kudryavtsev, and I. A. Movchan, Zh. Tekh. Fiz. **62**, 68 (1992) [Sov. Phys. Tech. Phys. **37**, 1004 (1992)].
[13] R. R. Arslanbekov, A. A. Kudryavtsev, and I. A. Movtchan, IEEE Trans. Plasma Sci. **24**, 1079 (1996).
[14] R. R. Arslanbekov, R. C. Tobin, and A. A. Kudryavtsev, J. Appl. Phys. **81**, 554 (1997).
[15] Z. Donkó, Phys. Rev. E **57**, 7126 (1998).
[16] C. J. Elliott and A. E. Greene, J. Appl. Phys. **47**, 2946 (1976).
[17] J. Bretagne, J. Godart, and V. Puech, J. Phys. D **15**, 2205 (1982).
[18] N. B. Kolokolov, A. A. Kudryavtsev, and V. A. Romanenko, Fiz. Plasmy **15**, 612 (1989) [Sov. J. Plasma Phys. **15**, 357 (1989)].
[19] M. A. Lieberman and S. Ashida, Plasma Sources Sci. Technol. **5**, 145 (1996).
[20] J. C. Ingraham and S. C. Brown, Phys. Rev. **138**, A1015 (1965).
[21] A. B. Blagoev, Y. M. Kagan, N. B. Kolokolov, and R. I. Lyagushchenko, Zh. Tekh. Fiz. **19**, 215 (1974) [Sov. Phys. Tech. Phys. **19**, 215 (1974)].

[22] N. B. Kolokolov, A. A. Kudryavtsev, and O. G. Toronov, Zh. Tekh. Fiz. **30**, 1128 (1985) [Sov. Phys. Tech. Phys. **30**, 1128 (1985)].
[23] N. B. Kolokolov, A. A. Kudryavtsev, and V. A. Romanenko, Zh. Tekh. Fiz. **33**, 1274 (1988) [Sov. Phys. Tech. Phys. **33**, 1274 (1988)].
[24] N. B. Kolokolov, A. A. Kudrjavtsev, and A. B. Blagoev, Phys. Scr. **50**, 371 (1994).
[25] A. A. Kudryavtsev and A. I. Ledyankin, Phys. Scr. **53**, 597 (1996).
[26] J. Bretagne, G. Delouya, M. Capitelli, C. Gorse, and M. Bacal, J. Phys. D **19**, 1197 (1986).
[27] G. Lister, J. Vac. Sci. Technol. A **14**, 2736 (1996).
[28] M. Surendra, D. B. Graves, and G. M. Jellum, Phys. Rev. A **41**, 1112 (1990).
[29] A. Fiala, L. C. Pitchford, and J. P. Boeuf, Phys. Rev. E **49**, 5607 (1994).
[30] J. V. DiCarlo and M. J. Kushner, J. Appl. Phys. **66**, 5763 (1989).
[31] T. J. Sommerer, W. N. G. Hitchon, and J. E. Lawler, Phys. Rev. A **39**, 6356 (1989).
[32] D. B. Graves and K. F. Jensen, IEEE Trans. Plasma Sci. **14**, 78 (1986).
[33] P. Bayle, J. Vacquie, and M. Bayle, Phys. Rev. A **34**, 360 (1986).
[34] R. R. Arslanbekov, A. A. Kudryavtsev, Y. A. Piotrovskii, and Y. A. Tolmachev, High Temp. **32**, 153 (1994).
[35] R. R. Arslanbekov and A. A. Kudryavtsev, in *Electron Kinetics and Applications of Glow Discharges*, Vol. 367 of *NATO Advanced Study Institute Series B: Physics*, edited by U. Kortshagen and L. D. Tsendin (Plenum, New York, 1998), pp. 161–178.
[36] T. Bräuer, S. Gortchakov, D. Loffhagen, S. Pfau, and R. Winkler, J. Phys. D **30**, 3223 (1997).
[37] A. Dalgarno, M. B. McElroy, and R. J. Moffett, Planet. Space Sci. **11**, 463 (1963).
[38] I. A. Krinberg, Planet. Space Sci. **21**, 523 (1973).
[39] R. R. Arslanbekov, A. A. Kudryavtsev, and I. A. Movchan, Zh. Tekh. Fiz. **62**, 7 (1992) [Sov. Phys. Tech. Phys. **37**, 620 (1992)].
[40] L. D. Tsendin, in *Electron Kinetics and Applications of Glow Discharges*, Vol. 367 of *NATO Advanced Study Institute Series B: Physics*, edited by U. Kortshagen and L. D. Tsendin (Plenum, New York, 1998), pp. 1–18.
[41] L. D. Tsendin, Zh. Eksp. Teor. Fiz. **66**, 1638 (1974) [Sov. Phys. JETP **39**, 805 (1974)].
[42] L. D. Tsendin, Plasma Sources Sci. Technol. **4**, 200 (1995).
[43] V. I. Kolobov and V. A. Godyak, IEEE Trans. Plasma Sci. **23**, 503 (1995).
[44] U. Kortshagen, C. Busch, and L. D. Tsendin, Plasma Sources Sci. Technol. **5**, 1 (1996).
[45] V. E. Golant, A. P. Zhilinsky, and I. E. Sakharov, *Fundamentals of Plasma Physics* (Wiley, New York, 1980).
[46] L. D. Tsendin and Y. B. Golubovskii, Zh. Tekh. Fiz. **47**, 1839 (1977) [Sov. Phys. Tech. Phys. **22**, 1066 (1977)].
[47] Y. B. Golubovskii, Y. M. Kagan, and R. I. Lyagushchenko, Zh. Eksp. Teor. Fiz. **57**, 2222 (1969) [Sov. Phys. JETP **30**, 1204 (1970)].
[48] A. P. Zhilinskii, I. F. Liventseva, and L. D. Tsendin, Zh. Tekh. Fiz. **47**, 304 (1977) [Sov. Phys. Tech. Phys. **22**, 177 (1977)].
[49] P. M. Chung, Phys. Fluids **12**, 1623 (1969).
[50] S. D. Rockwood, Phys. Rev. A **8**, 2348 (1973).
[51] D. U. von Rosenberg, *Methods for the Numerical Solution of Partial Differential Equations* (Elsevier, New York, 1969).